Stochastic Calculus
Paris Dauphine University - Master IEF (272)

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Exercises Chapter 4

Exercise 1 *What is the price of a European Call option on a non-dividend paying stock when the stock price is 26€, the strike price is 25€, the risk-free interest rate is 10% per annum, the volatility is 30% per annum, and the time to maturity is three months?*

Exercise 2 *What is the price of a European Put option on a non-dividend paying stock when the stock price is 69€, the strike price is 70€, the risk-free interest rate is 5% per annum, the volatility is 35% per annum, and the time to maturity is six months?*

Exercise 3 *Assume that a non-dividend paying stock has an expected return of μ and a volatility of σ with the log return of the stock price been normally distributed.*

Prove that a 95% confidence interval for S_T is given by $\left(S_0 e^{\left(\mu-\frac{1}{2}\sigma^2\right)T-1.96\sigma\sqrt{T}}; S_0 e^{\left(\mu-\frac{1}{2}\sigma^2\right)T+1.96\sigma\sqrt{T}}\right)$.

Exercise 4 *Assume that a non-dividend paying stock has an expected return of μ and a volatility of σ with the log return of the stock price been normally distributed.*

A financial institution has just announced that it will trade a derivative that pays off an euro amount equal to $\ln S_T$ at time T where S_T denotes the values of the stock price at time T.

a) What is the price, f, of the derivative at time t in term of the stock price, S, at time t according to a risk-neutral valuation? (We denote by r the risk-free interest rate.)

b) Verify that your price satisfies the Black-Scholes-Merton differential equation:

$$\frac{\partial f}{\partial t} + rS\frac{\partial f}{\partial S} + \frac{\sigma^2}{2}S^2\frac{\partial^2 f}{\partial S^2} = rf$$