Master PEI: Game Theory in the International Arena
Answer to the Final Exam, Sciences Po, December 2017

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Paris and Frankfurt compete to woo Britain's banks post-Brexit (16 pts).
Part A. Competition between Paris and Frankfurt (9 pts).

A1. (**3 pts**) The corresponding matrix payoff writes as

		FM's choice	
		H	L
P's choice	H	$(\frac{1}{2}-c_H;\frac{1}{2}-c_H)$	$(1-c_H;0)$
	L	$(0;1-c_H)$	$(\frac{1}{2};\frac{1}{2})$

From $c_H < \frac{1}{2}$ we have $\frac{1}{2} - c_H > 0$ and $1 - c_H > \frac{1}{2}$, so H is a strictly dominant strategy for every city. The set of pure strategy Nash equilibrium is the singleton $\{(H, H)\}$.

The outcome (H, H) is Pareto-dominated by (L, L).
From $c_H < \frac{1}{2}$ we have $1 - c_H > \frac{1}{2}$, so the outcome (H, L) gives P its (unique) maximal payoff. Hence, the outcome (H, L) is Pareto-efficient.
Similarly, the outcome (L, H) gives FM its (unique) maximal payoff. Hence, the outcome (L, H) is Pareto-efficient.
Finally, P is strictly better off under (L, L) than under (L, H), while FM is strictly better off under (L, L) than under (H, L), so (L, L) is Pareto-efficient as well.
Therefore, the set of Pareto-efficient outcomes is $\{(H, L); (L, H); (L, L)\}$.

A2. (**3 pts**) The corresponding matrix payoff writes as

		FM's choice	
		F	R
P's choice	F	$(\frac{1}{2};\frac{1}{2})$	$(1;0)$
	R	$(0;1)$	$(\frac{1}{2};\frac{1}{2})$

Clearly, F is a strictly dominant strategy for every city. The set of pure strategy Nash equilibrium is the singleton $\{(F, F)\}$.

The outcome (F, R) gives P its (unique) maximal payoff. Hence, the outcome (F, R) is Pareto-efficient.
Similarly, the outcome (R, F) gives FM its (unique) maximal payoff. Hence, the outcome (R, F) is Pareto-efficient.
P is strictly better off under (R, R) than under (R, F), while FM is strictly better off under (R, R) than under (F, R), so (R, R) is Pareto-efficient as well.
Finally, since the outcome (F, F) provides the same payoff than does (R, R), (F, F) is Pareto-efficient as well.
Therefore, the set of Pareto-efficient outcomes is $\{(F, F); (F, R); (R, F); (R, R)\}$.

A3. (**3 pts**) The corresponding matrix payoff writes as

		FM's choice			
		H,F	H,R	L,F	L,R
P's choice	H,F	$(\frac{1}{2}-c_H;\frac{1}{2}-c_H)$	$(1-c_H;-c_H)$	$(1-c_H;0)$	$(1-c_H;0)$
	H,R	$(-c_H;1-c_H)$	$(\frac{1}{2}-c_H;\frac{1}{2}-c_H)$	$(1-\alpha-c_H;\alpha)$	$(1-c_H;0)$
	L,F	$(0;1-c_H)$	$(\alpha;1-\alpha-c_H)$	$(\frac{1}{2};\frac{1}{2})$	$(1;0)$
	L,R	$(0;1-c_H)$	$(0;1-c_H)$	$(0;1)$	$(\frac{1}{2};\frac{1}{2})$

From $0<c_H<\frac{1}{2}$ we have $\frac{1}{2}-c_H>0>-c_H$, so $BR^{FM}\,(P \text{ plays } (H,F))=\{(H,F)\}$.
From $0<c_H<\frac{1}{2}$ and $\alpha<1-c_H$ we have $1-c_H>\max\{\frac{1}{2}-c_H;\alpha;0\}$, so $BR^{FM}\,(P \text{ plays } (H,R))=\{(H,F)\}$.
From $c_H<\frac{1}{2}$ and $\alpha>0$ we have $1-c_H>\max\{1-\alpha-c_H;\frac{1}{2};0\}$, so $BR^{FM}\,(P \text{ plays } (L,F))=\{(H,F)\}$.
From $0<c_H$ we have $BR^{FM}\,(P \text{ plays } (L,R))=\{(L,F)\}$.
By symmetry, the correspondence of P's best response are the same. Hence, $\{(H,F),(H,F)\}$ is a Nash equilibrium and it is the unique pure strategy equilibrium.
This Nash equilibrium is Pareto-dominated by the outcome $\{(L,R),(L,R)\}$.

Part B. European harmonization of tax policies (3 pts).

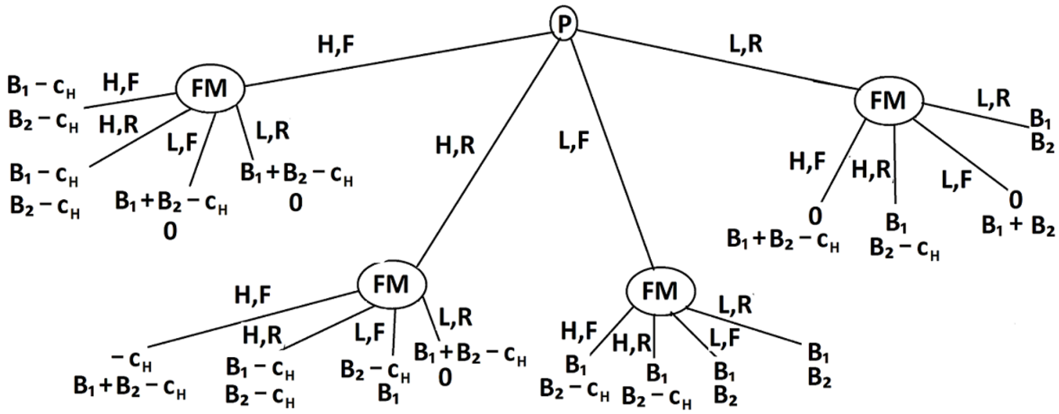
B1. (**3 pts**) By choosing one of the two levels of tax cuts (H or L) the European Union transforms the previous game depicted in question A3, into a 2×2 matrix game. If the EU chooses H the game writes as

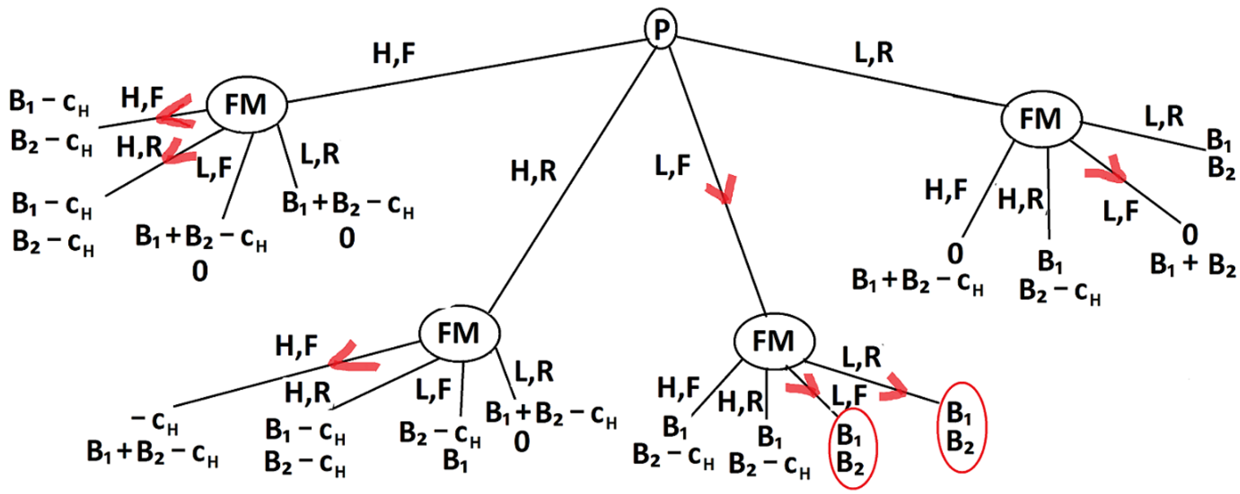
		FM's choice	
		F	R
P's choice	F	$(\frac{1}{2}-c_H;\frac{1}{2}-c_H)$	$(1-c_H;-c_H)$
	R	$(-c_H;1-c_H)$	$(\frac{1}{2}-c_H;\frac{1}{2}-c_H)$

This game has a unique Nash equilibrium: $\{(F,F)\}$, with a corresponding payoff $(\frac{1}{2}-c_H;\frac{1}{2}-c_H)$.
If the EU chooses L, the game writes as

		FM's choice	
		F	R
P's choice	F	$(\frac{1}{2};\frac{1}{2})$	$(1;0)$
	R	$(0;1)$	$(\frac{1}{2};\frac{1}{2})$

This game has a unique Nash equilibrium: $\{(F,F)\}$, with a corresponding payoff $(\frac{1}{2};\frac{1}{2})$.
From $0<c_H$ the Nash equilibrium of the first game (when H is chosen) is Pareto-dominated by the one of the second game (when L is chosen), so the level of tax cuts that should be selected by the European Commission is the lowest one: L.





Part C. City banks (4 pts).

C1. (**2 pts**) The game tree where Paris plays first and Frankfurt move second writes as:

Figure 1

C2. (**2 pts**) From $B_2 - c_H > 0$ the set of subgame perfect Nash equilibria is easily obtain by backward induction:

Figure 2

It contains two equilibria and writes as $\{((L,F),(L,F);((L,F),(L,R)))\}$. The payoffs associated with any of these two equilibria is the same: (B_1, B_2). It consists of Paris attracting the first group of banks with a low tax cut and a flexible hiring-and-firing regime (i.e., (L,F)); and for Frankfurt to attract the second group of banks with a low tax cut and any kind of hiring-and-firing regime (i.e., (L,F) or (L,R)).

Questions (4 pts)

Are the following statements correct? If not, give a counter-example.

Q1. (**2 pts**) The statement that the manner in which people discount future payoffs is the same for everyone is false. For instance, in chapter 3, we saw that according to the paper of Harrison, Lau and Williams (AER 2002), poor (resp. less educated) people have a lower discount factor than rich (resp. more educated) people, with the interpretation that they are less patient.

Q2. (**2 pts**) The statement that the manner in which an individual discounts future payoffs is the same among all periods is false. For instance, in chapter 3, we saw that in a study conducted by Read and van Leeuwen (1998), most involved people had a strong preference for the immediate present. Namely, in response to the question: “If choosing today would you choose fruit or chocolate for next week?” 74 % chose fruit; while to the question: “For today, what do you choose?” 70 % chose chocolate.